Povzetek

V diplomskem delu predstavimo geometrijo evklidskih trikotnikov, izraženo s kompleksnim križnim razmerjem. Najprej pokažemo, da je vsak trikotnik do podobnosti natanko določen z enim samim kompleksnim številom, imenovanim oblika trikotnika. S pomočjo oblik in osnovnih dveh izrekov o obliki dokažemo izreke, ki obravnavajo podobne trikotnike. Nato z uporabo križnega razmerja koordinatiziramo evklidsko ravnino in uporabimo trikotniške koordinate za dokazovanje izrekov o trikotnikih. Pokažemo še kompleksni verziji Menelajevega izreka in Cevovega izreka. Na koncu si ogledamo trikotniške koordinate posebnih točk trikotnika in pokažemo, da so le-te funkcije oblike trikotnika. Pokažemo še, kako lahko te funkcije uporabimo pri dokazovanju nekaterih izrekov o trikotnikih.

Ključne besede: evklidska ravnina, kompleksna ravnina, trikotnik, oblike, Miquelovi trikotniki, trikotniške koordinate, trikotniške funkcije, Morleyev izrek, Napoleonov izrek, Cevov izrek, Menelajev izrek.

Abstract

In the first part we examine Euclidean triangle geometry via complex cross ratio. We show that every triangle can be characterized up to similarity by a single complex number, called its shape. We then use shapes and two basic theorems about similar triangles. We also coordinatize the Euclidean plane using cross ratios, and use these triangle coordinates to prove theorems about triangles. We develop complex versions of the theorems of Ceva and Menelaus. At the end we look at the triangle coordinates of special points of a triangle, and show that they are functions of its shape. We also show how these functions can be used to prove theorems about triangles.

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Keywords: Euclidean plane, complex plane, triangle, shapes, Miquel triangles, triangle coordinates, triangle functions, Morley's theorem, Napoleon's theorem, Ceva's theorem, Menelaus's theorem.

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