Povzetek

V prvem poglavju diplomskega dela najprej navedemo nekaj osnovnih definicij in lastnosti polinomov več spremenljivk. V razdelku 1.2 obravnavamo afine algebraične krivulje ter opišemo nekaj primerov singularnih točk. Nato afino ravnino $\mathbb{C}^2$ vložimo v projektivno ravnino $\mathbb{P}_2(\mathbb{C})$ in definiramo presečne večkratnosti projektivnih algebraičnih krivulj. Na koncu zapišemo Bézoutov izrek.

V drugem poglavju izpeljemo Plückerjeve formule za projektivno algebraično krivuljo $\mathcal{C}$. V razdelku 2.1 s pomočjo polare iz dane točke konstruiramo tangente na gladke točke krivulje $\mathcal{C}$. S tem določimo razred krivulje. Nato s Hessejevo krivuljo ocenimo število prevojev. V razdelku 2.3 vpeljemo pojem dualne krivulje $\mathcal{C}^*$ in nekaj zvez med $\mathcal{C}$ in $\mathcal{C}^*$. Na koncu definiramo Plückerjeve krivulje in za njih izpeljemo Plückerjeve formule.

V tretjem poglavju najprej dokažemo, da ima vsaka nerazcepna in nesingularna projektivna kvartika natanko 28 bitangent. Nato definiramo Steinerjeve in Aronholdove množice bitangent in opišemo njihove lastnosti. V razdelku 3.2 vpeljemo pojem realne algebraične krivulje in raziščemo, koliko realnih bitangent ima realna kvartika. Narišemo kvartiko, pri kateri so vse bitangente realne. Na koncu še zapišemo enačbo krivulje, ki dano kvartiko seka v dotikališčih njenih bitangent.

V zadnjem poglavju izpeljemo enačbe 28 bitangent Kleinove kvartike. Delo zaključimo s kratkim opisom konfiguracije teh bitangent.

Vsi novi geometrijski pojmi so ponazorjeni na primerih ravninskih kvartik. Algebraični izračuni in večina slik je narejenih v programu Mathematica. Konec dokaza je označen s $\square$, konec primera pa s $\triangle$.

Math. Subj. Class. (2000): 51N35, 14H50, 14H45, 12E05

Ključne besede: afina algebraična krivulja, projektivna algebraična krivulja, polara, Hessejeva krivulja, dualna krivulja, bitangenta, Plückerjeve formule, ravninske kvartike, Steinerjeva množica, Aronholdova množica, Trottova krivulja, Kleinova kvartika.

Abstract

We begin the first chapter of this diploma thesis with some basic definitions and properties of polynomials with multiple variables. In section 1.2 we discuss affine algebraic curves and describe some possible types of singular points. We then embed the affine plane $\mathbb{C}^2$ in the projective plane $\mathbb{P}_2(\mathbb{C})$ and define intersection multiplicities of projective algebraic curves. At the end we write down Bézout's theorem.

In chapter 2 we derive Plücker formulas for the projective algebraic curve $\mathcal{C}$. In section 2.1 we use polar to construct tangents to smooth points of the curve $\mathcal{C}$ that pass through a prescribed point. With this we determine the class of the curve. With Hessian curve we estimate the number of inflection points. In section 2.3 we introduce the dual curve $\mathcal{C}^*$ and some relationships between $\mathcal{C}$ and $\mathcal{C}^*$. At the end we define Plücker curves and derive Plücker formulas for them.

In chapter 3 we first prove, that every irreducible and non-singular projective quartic curve has exactly 28 bitangents. Then we define Steiner and Aronhold sets and describe their properties. In section 3.2 we introduce real algebraic curves and explore how many real bitangents a real quartic has. We draw a quartic with all 28 real bitangents. At the end we write down the equation of the curve that intersects a given quartic in the points of tangency of its bitangents.

In the last chapter we derive the equations of 28 bitangents of Klein quartic. The thesis is concluded with short description of their configuration.

All new geometrical terms are illustrated through examples of plane quartics. Algebraic calculations and most figures are made by Mathematica program. The end of the proof is marked with $\square$, the end of example with $\triangle$.

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Keywords: affine algebraic curve, projective algebraic curve, polar, Hessian curve, dual curve, bitangent, Plücker formulas, plane quartics, Steiner set, Aronhold set, Trott curve, Klein quartic.

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