Povzetek

Predstavljen je kvadratni problem lastnih vrednosti $(\lambda^2 A + \lambda B + C)x = 0$ in njegova pomembna vloga pri reševanju linearne diferencialne enačbe drugega reda. Podrobneje obravnavamo kvadratni problem lastnih vrednosti s Hermitskimi matrikami A, B in C ter pozitivno definitno matriko A. Hermitski kvadratni problem lastnih vrednosti ima dva pomembna podrazreda, hiperbolični in eliptični problem. Prvi ima realne in drugi kompleksne lastne vrednosti. Hiperbolični problem z nepozitivnimi lastnimi vrednostmi imenujemo nadkritično dušen. Minimaks izrek, ki ga poznamo za linearni problem lastnih vrednosti, lahko izpeljemo tudi za hiperbolični problem.
Osredotočili smo se na preverjanje hiperboličnosti Hermitskega kvadratnega problema lastnih vrednosti. Izpeljana sta dva algoritma za preverjanje te lastnosti. Pri prvem tvorimo par matrik (M, N), ki ga konstruiramo iz matrik A, B, C podanega Hermitskega problema lastnih vrednosti. Algoritem temelji na preverjanju definitnosti para matrik. Drugi algoritem je iteracija za preverjanje nadkritične dušenosti kvadratnega problema lastnih vrednosti. Za hiperbolični problem velja, da ga s primernim premikom spektra lahko preoblikujemo v nadkritično dušenega.
Z numeričnimi primeri je testirano delovanje algoritmov.

Math. Subj. Class. (2000): 65F15, 15A18, 15A22

Ključne besede: Kvadratni problem lastnih vrednosti, linearna diferencialna enačba drugega reda, Hermitski kvadratni problem lastnih vrednosti, hiperbolični problem, eliptični problem, nadkritično dušen problem, minimaks izrek, definiten par, dušen sistem mas in vzmeti.

Abstract

We present the quadratic eigenvalue problem $(\lambda^2 A + \lambda B + C)x = 0$ and its important role at solving second-order linear differential equations. In particular we treat quadratic eigenvalue problems with Hermitian matrices A, B, C and a positive definite matrix A. There are two important classes of Hermitian eigenvalue problem, hyperbolic and elliptic problems. A hyperbolic problem has real eigenvalues and an elliptic problem has complex eigenvalues. A hyperbolic problem with nonpositive eigenvalues is called overdamped. Minimax theorem known for linear eigenvalue problem can be formed for hyperbolic problem too.
We have concentrated on testing hyperbolicity of Hermitian quadratic eigenvalue problem. Two algorithms for testing hyperbolicity are presented. The first algorithm forms matrix pair (M, N) from matrices A, B, C of a given Hermitian quadratic eigenvalue problem and then tests whether the pair (M, N) is a definite pair. The second algorithm is an iteration for testing the overdamping of a quadratic eigenvalue problem. We show that any hyperbolic problem can be transformed into an overdamped one by an appropriate shifting of the spectrum.
Numerical examples are given to illustrate algorithms.

Key words: Quadratic eigenvalue problem, second-order linear differential equation, Hermitian quadratic eigenvalue problem, hyperbolic problem, elliptic problem, overdamped problem, minimax theorem, definite pair, damped mass spring system.

Literatura

[1] R. J. Duffin, *A minimax theory for overdamped networks*, J. Rational Mech. Anal., **4** (1955), str. 221-233.

[2] I. Gohberg, P. Lancaster in L. Rodman, *Matrix Polynomials*, Academic Press, New York, 1982.

[3] C-H. Guo, N. J. Higham in F. Tisseur, *Detecting and solving hyperbolic quadratic eigenvalue problems*, MIMS Eprint: 2007.117, September 2007.

[4] C-H. Guo, P. Lancaster, *Algorithms for hyperbolic quadratic eigenvalue problems*, Math. Comp., **74** (2005), str. 1777-1791.

[5] Y. Hachez in P. Van Dooren, *Elliptic and hyperbolic quadratic eigenvalue problems and associated distance problems*, Linear Algebra Appl., **371** (2003), str. 31-44.

[6] N. J. Higham in F. Tisseur, *Bounds for Eigenvalues of Matrix Polynomials*, Linear Algebra Appl., **358** (2003), str. 5-22.

[7] N. J. Higham, F. Tisseur in P. M. Van Dooren, *Detecting a definite Hermitian pair and a hyperbolic or elliptic quadratic eigenvalue problem and associated nearness problems*, Linear Algebra Appl., **351-352** (2002), str. 455-474.

[8] M. E. Hochstenbach in H. A. Van Der Vorst, *Alternatives to the Rayleigh Quotient for the Quadratic Eigenvalue Problem*, SIAM J. Sci. Comput., **25** (2003), str. 591-603.

[9] P. Lancaster, *Lambda-Matrices and Vibrating Systems*, Pergamon Press, Oxford, UK, 1966.

[10] A. S. Markus, *Introduction to the spectral theory of polynomial operator pencils*, Amer. Math. Soc., Providence, 1988.

[11] B. Plestenjak, *Numerical methods for the tridiagonal hyperbolic quadratic eigenvalue problem*, SIAM J. Matrix Anal. Appl., **28** (2006), str. 1157-1172.

[12] B. Plestenjak, *Numerično reševanje hiperboličnega kvadratnega problema lastnih vrednosti*, Obzornik za matematiko in fiziko, **51** (2004), str. 65-78.

[13] G. W. Stewart in J.-G. Sun, *Matrix perturbation theory*, Academic Press, Inc., Boston, 1990.

[14] F. Tisseur in K. Meerbergen, *The quadratic eigenvalue problem*, SIAM Review, **43** (2001), str. 235-286.

[15] H. Voss, *A maxmin principle for nonlinear eigenvalue problems with application to a rational spectral problem in fluid-solid vibration*, Applications of Mathematics, **48** (2003), str. 607-622.