Bolyai-Gerwienov izrek in tretji Hilbertov problem

Povzetek

David Hilbert je leta 1900 predstavil seznam triindvajsetih nerešenih matematičnih problemov. Tretji problem na njegovem seznamu postavlja vprašanje, ali je mogoče poljubna dva poliedra enake prostornine vedno tako razdeliti na končno mnogo poliedrov, da prvi polieder sestavimo v drugega?

Hilbert je predvideval, da je odgovor na to vprašanje negativen, kar je še istega leta potrdil njegov študent Max Dehn s protiprimerom. Odgovor na podobno vprašanje o večkotnikih v dveh dimenzijah, to je Bolyai-Gerwienov izrek, je pozitiven in je bil znan že dolgo časa.

Bolyai-Gerwien theorem and Hilbert's third problem

Abstract

In 1900 David Hilbert presented a list of twenty-three unsolved mathematical problems. The third on his list is related to the following question: given any two polyhedra of equal volume, is it always possible to cut the first into finitely many polyhedral pieces, which can be reassembled to yield the second?

Hilbert conjectured that the answer is negative, which was confirmed within a year by his student Max Dehn, who proved that by presenting a counterexample. The answer for the analogous question about polygons in two dimensions is positive and had been known for a long time, this is the Bolyai-Gerwien theorem.

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Ključne besede: večkotnik, ploščina, polieder, prostornina, razdelitev, enakost po razdelitvi, Dehnova invarianta, enakost po dopolnitvi

Keywords: polygon, area, polyhedron, volume, dissection, equidissectable, Dehn invariant, equicomplementable

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