

## Povzetek

V vsakdanjem življenju se srečujemo z različnimi vrstami mehanizmov in sklopov, ki nam lajšajo delo in gibanje. Mehanizme so uporabljali že stari Grki, za matematike pa so postali posebej zanimivi v 19. stoletju, ko je Kempe predstavil svoj univerzalnostni izrek. Izrek pravi, da za poljubno 'polinomske krivulje' obstaja sklop, ki jo nariše. V diplomskem delu osvetlimo zgodovino mehanizmov in sklopov ter dokažemo Kempejev izrek. V nadaljevanju s programom GeoGebra konstruiramo opisane sklope.

## Abstract

In everyday life we are met with various types of mechanisms and linkages which ease our work and movement. These mechanisms were used by the ancient Greeks, but for mathematicians they became interesting in the 19<sup>th</sup> century, when Kempe introduced his Universality Theorem. His theorem says that for any polynomial curve there is a linkage which draws it. This diploma paper sheds light on the history of mechanisms and linkages, and proves Kempe's theorem. In addition, with the GeoGebra program the previously mentioned linkages are constructed.

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**Ključne besede:** mehanizmi, sklopi, Kempejev univerzalnostni izrek, GeoGebra

**Keywords:** mechanisms, linkages, Kempe Universality Theorem, GeoGebra

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