Povzetek

V diplomskem delu sta obdelana dva tipa polarnega razcepa grup.

Prvi je posplošitev polarnega razcepa obrnljive kvadratne matrike na hermitsko in unitarno matriko. Grupa G, skupaj z antiavtomorfno involucijo $f : G \to G$, je *polarna*, če za vsak element $x \in G$ obstaja koren $r := \sqrt{f(x)x}$, za katerega velja $f(r) = r$. V taki grupi lahko za poljuben element $x \in G$ naredimo *polarni razcep* $x = ab$, kjer je $f(a)a = e$ in $f(b) = b$.

Natančneje sta obdelana primera, ko za grupo vzamemo obrnljive kompleksne kvadratne matrike, za involucijo pa enkrat adjungiranje, drugič transponiranje. Za poljubno (lahko tudi neobrnljivo) matriko je podan potreben in zadosten pogoj za obstoj polarnega razcepa. Dodan je še polarni razcep omejenega linearnega operatorja na Hilbertovem prostoru, ki nam ponuja definicijo razcepa za polgrupe z enoto.

V zadnjem poglavju je opisan drug tip polarnega razcepa. Vsak avtomorfizem u polne stopničaste filtrirane (asociativne ali Liejeve) algebre nad poljem s karakteristiko nič lahko razcepimo v obliki $u = u_0\exp(d)$, kjer je u_0 avtomorfizem, ki ohranja stopnice, d pa odvajanje $d : A \to A$.

Abstract

Two types of a polar decomposition of a group are presented.

The idea for the first one comes from linear algebra. Any square complex matrix A can be factorized as $A = UH$, where U is unitary and H is self-adjoint. A group G together with an antiautomorphic involution $f : G \to G$ is called *a polar group* if for any $x \in G$ there exists a symmetric square root $r := \sqrt{f(x)x}$, such that $r = f(r)$. Then every $x \in G$ can be factorized as $x = ab$, where $af(a) = e$ and $f(b) = b$.

Two special cases when $G = \mathrm{GL}(n, \mathbb{C})$ are treated, the case when the involution is given by $f(X) = X^*$ and the one when $f(X) = X^\mathsf{T}$. Neccessary and sufficient conditions are given for the existence of a polar decomposition for any (perhaps singular) square complex matrix. The polar decomposition of a bounded linear operator on a Hilbert space gives us an idea how to decompose semigroups with unit.

In the last chapter the second type is discussed. Any automorphism u of a complete graded filtered (associative or Lie) algebra over a field of characteristic zero can be factorized as $u = u_0\exp(d)$, where u_0 is an automorphism that preserves degree and $d : A \to A$ is a filtration increasing derivation.

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