Povzetek

V diplomskem delu najprej uvedemo in nato obravnavamo nekatere lastnosti križnega produkta.

V prvem poglavju so navedene nekatere osnovne definicije in lastnosti von Neumannovih algeber, ki jih nato potrebujemo v naslednjih poglavjih. Večino dokazov teh trditev opustimo.

V naslednjem poglavju trojici (M, G, α), kjer je M von Neumannova algebra, G lokalno kompaktna topološka grupa in α zvezno delovanje G na M, priredimo novo von Neumannovo algebro, ki jo imenujemo križni produkt von Neumannove algebre M glede na delovanje α grupe G in jo označimo z $M \otimes_\alpha G$.

V tretjem poglavju najprej definiramo delovanje θ grupe G na von Neumannovi algebri $M \otimes \mathcal{B}(L^2(G))$ in nato pokažemo, da je križni produkt množica fiksnih točk v von Neumannovi algebri $M \otimes \mathcal{B}(L^2(G))$ za delovanje θ_t. V primeru, da je α prostorsko delovanje, znamo karakterizirati komutant von Neumannove algebre $M \otimes_\alpha G$.

Nazadnje obravnavamo še dualno strukturo križnega produkta. Omejimo se na primer, ko je grupa G komutativna. Kovariantnemu sistemu (M, G, α) priredimo nov kovarianten sistem $(\widehat{M}, \widehat{G}, \widehat{\alpha})$, pri čemer je von Neumannova algebra $\widehat{M}$ enaka $M \otimes_\alpha G$, $\widehat{\alpha}$ pa je zvezno delovanje dualne grupe $\widehat{G}$ na $M \otimes_\alpha G$.

Abstract

Some basic properties of crossed products are studied in this report.

First of all, some basic definitions, and some properties, but mostly without proofs, of von Neumann algebras that are used later on, are given.

In Chapter 2 a new von Neumann algebra, which is called a crossed product of the von Neumann algebra M according to the action α of the group G, and which we put down as $M \otimes_\alpha G$, is associated to the triple (M, G, α), where M is von Neumann algebra, G locally compact topological group, and α a continuous action of G on M.

In the next part we define an action θ of the group G on von Neumann algebra $M \otimes \mathcal{B}(L^2(G))$ and then we show that the crossed product is a set of fixed points in a von Neumann algebra $M \otimes \mathcal{B}(L^2(G))$ for the action θ_t. If α is spatial we can also characterize the commutant of the von Neumann algebra $M \otimes_\alpha G$.

Finally, in the last chapter, some dual structures of the crossed product are treated. We study the case when the group G is commutative. Then we associate with the covariant system (M, G, α) a new one $(\widehat{M}, \widehat{G}, \widehat{\alpha})$, where von Neumann algebra $\widehat{M}$ is equal to $M \otimes_\alpha G$ and $\widehat{\alpha}$ is continuous action of dual group $\widehat{G}$ on $M \otimes_\alpha G$.

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