Povzetek

Preslikavo $f: E \to F$ med Banachovima prostoroma E in F, za katero velja $| \|f(x) - f(y)\| - \|x - y\| | \leq \varepsilon$, za neko pozitivno realno število ε in poljubni točki x in y iz E, imenujemo ε-izometrija. Naj bosta E in F končno razsežna normirana prostora, iste razsežnosti. Tedaj obstaja taka afina izometrija $v: E \to F$, da za vsako točko x iz E velja $\|v(x) - f(x)\| \leq 2\varepsilon$. V posebnem primeru, ko preslikava f slika 0 v 0, pa je izometrija v linearna.

Če prostora E in F nista končno razsežna, preslikava f pa je surjektivna, lahko prav tako najdemo afino izometrijo v, ki jo enakomerno aproksimira, vendar moramo oceno za razliko omiliti na $\|v(x) - f(x)\| \leq 5\varepsilon$, za vsako točko x iz E. Če pa je množica točk iz E, v katerih je norma odvedljiva v Frechetovem smislu, gosta v E, pa spet velja ocena $\|v(x) - f(x)\| \leq 2\varepsilon$, za vsako točko x iz E.

Abstract

A mapping $f: E \to F$ between Banach spaces E and F such that, for each pair of points x and y in E the inequality $| \|f(x) - f(y)\| - \|x - y\| | \leq \varepsilon$ is satisfied, is called an ε-isometry. Let E and F be normed spaces of the same finite dimension. There exists an affine isometry v, such that inequality $\|v(x) - f(x)\| \leq 2\varepsilon$ is satisfied for each x in E. In the special case, when f sends 0 to 0, is v linear isometry.

If E and F are not finite dimensional, but f is surjective, we can also find an affine isometry, that approximates f uniformly, but in this case we can only prove that $\|v(x) - f(x)\| \leq 5\varepsilon$. In another special case, when the set of points in E at which the norm is Frechet differentiable, is dense in E, the inequality $\|v(x) - f(x)\| \leq 2\varepsilon$ is again satisfied.

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