Povzetek

Naj bo $\mathfrak{G}$ rotacijsko invariantna družina gladkih Jordanskih krivulj, vsebovanih v odprtem enotskem krogu Δ v kompleksni ravnini. Za vsak $\Gamma \in \mathfrak{G}$ označimo z D_Γ enostavno povezano območje, omejeno z Γ. Spoznamo več pogojev, iz katerih sledi: če je f zvezna funkcija na Δ, taka, da ima za vsak $\Gamma \in \mathfrak{G}$ funkcija $f|_\Gamma$ zvezno razširitev na $\overline{D_\Gamma}$, analitično v D_Γ, tedaj je f analitična v Δ. Pri tem obravnavamo več primerov, in sicer glede na lego izhodišča. Pomagamo si s Fourierovo analizo, ko iščemo zvezo med obstojem razširitev in koeficienti Laurentovega razvoja funkcije. Rezultate zaokrožimo z različnimi zgledi.

Abstract

Let $\mathfrak{G}$ be a rotation invariant family of smooth Jordan curves contained in the open unit disc Δ in the complex plane. For each $\Gamma \in \mathfrak{G}$ denote by D_Γ the simply connected domain bounded by Γ. We present various conditions which imply that if f is a continuous function on Δ such that for every $\Gamma \in \mathfrak{G}$ the function $f|_\Gamma$ has a continuous extension to $\overline{D_\Gamma}$ which is analytic in D_Γ, then f is analytic in Δ. We divide the general situation into several cases, with respect to the position of the origin. We use Fourier analysis when searching the connection between the existence of the extensions and Laurent coefficients of the function. The results are illustrated by various examples.

Math. Subj. Class. (1991): 31A20, 40G05.

Key words: holomorphic extensions, rotation invariance, Fejér theorem, algebras of holomorphic functions.

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