

Povzetek

V diplomskem delu je pokazano, da ne obstaja noben povezavno kritičen graf reda 14. Vprašanje najmanjšega protiprimera za domnevo o kritičnih grafih [2, 18] ostaja tako odprto le še za red 16, saj ima najmanjši znani protiprimer red 18 [26]. Drugi rezultat pričujočega dela je konstrukcija vseh netrivialnih kritičnih grafov reda 13. Vsi so 3-kritični in imajo za minor Petersenov graf brez enega vozlišča.

Vsak Δ -kritičen graf reda n je podgraf Δ -maksimalnega grafa reda n . Tako so bili generirani vsi zanimivi Δ -maksimalni grafi, nato pa je bilo z metodo rekurzivnega kleščanja [5] preverjeno, če vsebujejo Δ -kritičen podgraf istega reda. Vsi najdeni kritični podgrafi so bili izpisani. Zaradi velikega števila grafov je bilo treba paziti, da se niso preverjali izomorfni primeri, pri rekurzivnem kleščanju pa se je bilo treba izogibati večkratnemu preverjanju istih podgrafov.

Ključne besede: kritični graf, barvanje povezav, barvanje grafa, generiranje grafov.

Abstract

In this work we show that there are no chromatic-index-critical graphs of order 14. This yields that the question of smallest counterexample for the Critical graph conjecture [2, 18] is now open just for the order 16, since the smallest known counterexample has order 18 [26]. Next result is the construction of all nontrivial critical graphs of order 13. They are all 3-critical and have Petersen graph minus a vertex as their minor.

Each Δ -critical graph of order n is a subgraph of some Δ -maximal graph of order n . Thus all interesting Δ -maximal graphs were generated and then pruned with the method of recursive pruning [5] to see whether they contain a critical subgraph of the same order or not. In order to minimise the number of

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