

## Povzetek

Pričujoče diplomsko delo obravnava **visoko ločno tranzitivne digrafe**, to je tiste digrafe, katerih grupa avtomorfizmov deluje tranzitivno na pripadajočih množicah  $s$ -lokov za vsak  $s \geq 0$ . Pretežni del vsebine tega dela izhaja iz obsežnega raziskovalnega članka "Infinite highly arc transitive digraphs and universal covering digraphs", *Combinatorica*, **13** (1993), 377-396 (P. J. Cameron, C. E. Praeger in N. C. Wormald), predvsem pa se osredotoči na nekatere tam postavljene odprte probleme; tako tiste, ki so bili kasneje rešeni, kot tiste, ki ostajajo še naprej odprti. Prav pri enem od slednjih je to diplomsko delo tudi skromen, a izviren prispevek k obravnavani tematiki, saj pokaže, da ostaja eden od omenjenih problemov kljub nasprotni trditvi v članku "Descendants in highly arc transitive digraphs", *Discrete Math.*, **247** (2002), 147-157 (R. G. Möller) še naprej odprt.

Za povezan, 1-tranzitiven, bipartiten digraf  $\Delta$  se konstruira visoko ločno tranzitiven digraf  $DL(\Delta)$  in pokaže, da je digraf  $DL(\Delta)$  krovni digraf vsakega digrafa iz določenega razreda  $\mathcal{D}(\Delta)$  povezanih digrafov. Če je digraf  $\Delta$  tudi lokalno končen, je digraf  $DL(\Delta)$  celo univerzalni krovni digraf za razred  $\mathcal{D}(\Delta)$ .

Digraf  $D$  ima **lastnost  $Z$** , če obstaja kak epimorfizem digrafov iz digrafa  $D$  v digraf  $Z$ , to je digraf z množico točk  $\mathbb{Z}$  in množico lokov  $\{(i, i + 1) \mid i \in \mathbb{Z}\}$ . Pokaže se, da digraf  $DL(\Delta)$  ima lastnost  $Z$ . Odgovori se na nekatera vprašanja v zvezi z visoko ločno tranzitivnimi digrafi in lastnostjo  $Z$ , ki so bila postavljena v zgoraj omenjenem članku. Med drugim se konstruira visoko ločno tranzitivne digrafe iz razreda  $\mathcal{D}(\Delta)$ , ki nimajo lastnosti  $Z$ . Pokaže se, da ima vsak neskončen, povezan, visoko ločno tranzitiven digraf  $D$  s končno izhodno stopnjo, katerega izhodna razpršenost je enaka 1, lastnost  $Z$ . Če ima omenjeni digraf  $D$  tudi vhodno razpršenost enako 1, ima vsak homomorfizem digrafov iz digrafa  $D$  v digraf  $Z$  končna vlakna.

Math. Subj. Class. (2000): 05 C 05, 05 C 20, 05 C 25, 05 C 38, 05 C 70, 05 C 99

Ključne besede: digraf, grupa avtomorfizmov, visoko ločno tranzitiven, bipartiten, krovni digraf, univerzalni krovni digraf, lastnost  $Z$ , razpršenost, stopnja, končna vlakna

## Abstract

The present BSc thesis is concerned with **highly arc transitive digraphs**, that is, digraphs whose automorphism groups act transitively on the corresponding sets of  $s$ -arcs for all  $s \geq 0$ . The main part of this thesis covers a comprehensive research article "Infinite highly arc transitive digraphs and universal covering digraphs", *Combinatorica*, **13** (1993), 377-396 (P. J. Cameron, C. E. Praeger and N. C. Wormald). In particular, it focuses on some open problems stated there; the ones which have later been solved and also the ones that remain open. The above thesis is also a small but original contribution to this area of research as it shows that one of the above mentioned problems stays open in spite of a contrary statement made in the article "Descendants in highly arc transitive digraphs", *Discrete Math.*, **247** (2002), 147-157" (R. G. Möller).

For a connected, 1-arc transitive, bipartite digraph  $\Delta$ , a highly arc transitive digraph  $DL(\Delta)$  is constructed and is shown to be a covering digraph for every digraph in a certain class  $\mathcal{D}(\Delta)$  of connected digraphs. Moreover, if  $\Delta$  is locally finite, then  $DL(\Delta)$  is a universal covering digraph for  $\mathcal{D}(\Delta)$ .

A digraph  $D$  is said to have **property  $Z$** , if there exists a digraph epimorphism from  $D$  onto the digraph  $Z$  with vertex set  $\mathbb{Z}$  (the set of all integers) and the set of arcs  $\{(i, i+1) \mid i \in \mathbb{Z}\}$ . It is shown that  $DL(\Delta)$  has property  $Z$ . Furthermore, some of the questions concerning highly arc transitive digraphs and property  $Z$ , stated in the above mentioned article of Cameron, Praeger and Wormald, are answered. Also, highly arc transitive digraphs from class  $\mathcal{D}(\Delta)$  without property  $Z$  are constructed. It is shown that an infinite, connected, highly arc transitive digraph  $D$  with finite out-valency and out-spread 1 has property  $Z$ . If the in-spread of  $D$  also equals 1 then each digraph homomorphism from  $D$  to  $Z$  has finite fibres.

Math. Subj. Class. (2000): 05 C 05, 05 C 20, 05 C 25, 05 C 38, 05 C 70, 05 C 99

Keywords: digraph, automorphism group, highly arc transitive, bipartite, covering digraph, universal covering digraph,  $Z$  property, spread, valency, finite fibres

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