Povzetek

Operator A na Banachovem prostoru $L^p(X, \mu)$ je standardno trikotljiv, če obstaja veriga standardnih invariantnih podprostorov za operator A, ki je maksimalna kot veriga podprostorov prostora $L^p(X, \mu)$. V prvem poglavju dokažemo osnovne lastnosti o ascentu in descentu operatorja, katere potrebujemo za dokaz Rieszove dekompozicije kompaktnega operatorja. V drugem poglavju predstavimo osnovne lastnosti trikotljivosti ter dokažemo lemo o trikotljivosti, Ringroseov izrek in spektralni izrek za trikotljivo družino kompaktnih operatorjev. V poglavju o nenegativnih operatorjih dokažemo potrebne in zadostne pogoje, kdaj je polgrupa nenegativnih operatorjev standardno razcepna. V poglavju o integralskih operatorjih dokažemo, da na prostoru $L^p(X, \mu)$, kjer X nima atomov, identični operator ni integralski operator ter predstavimo osnovne rezultate o standardni trikotljivosti nenegativnih integralskih operatorjev. V 5. poglavju, iz matrik na operatorje s sledjo na separabilnem Hilbertovem prostoru, posplošimo definiciji sledi in determinante, znane formule, izreke in razvijemo razne kriterije za primerjanje lastnih vrednosti in singularnih vrednosti operatorja s sledjo. V zadnjem poglavju, kjer obravnavamo nenegativne operatorje s sledjo, podamo potrebne in zadostne pogoje, kdaj je nenegativen operator, ki je vsota identičnega operatorja in operatorja s sledjo, standardno trikotljiv.

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Ključne besede: kompaktni operatorji, nenegativni operatorji, integralski operatorji, operatorske polgrupe, invariantni podprostori, standardna trikotljivost, sled in determinanta

Abstract

An operator A on a Banach space $L^p(X, \mu)$ is standardly triangularizable if there exists a chain of standard invariant subspaces for operator A that is maximal as a subspace chain of subspaces of $L^p(X, \mu)$. In the first chapter, basic properties about ascent and descent of operator are proved which are needed for proof of Riesz decomposition theorem. In the second chapter, Triangularization lemma, Ringrose's theorem and Spectral theorem for triangularizable collection of compact operators are shown. In the chapter about non-negative operators necessary and sufficient conditions are given under which the semigroup of non-negative operators is standardly reducible. In the chapter about integral operators it is shown that the identity operator on $L^p(X, \mu)$ is not an integral operator when X is a continuous measure space. Basic properties about standard triangularizability of non-negative integral operators are also presented. In Chapter 5, the definitions of trace and determinant are generalized from matrices to the trace class operators on a separable Hilbert space, well known finite dimensional formulas are proved and many tools for comparing eigenvalues and singular values are developed. In the last chapter, non-negative trace class operators are studied, and necessary and sufficient conditions are given under which a non-negative operator of form identity plus trace class is standardly triangularizable.

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Keywords: compact operators, nonnegative operators, integral operators, operator semigroups, invariant subspaces, standard triangularization, trace and determinant

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