

Povzetek

V tem delu bomo obravnavali sferične t -načrte v realnem evklidskem prostoru. Z uporabo lastnosti harmoničnih in homogenih polinomov, ki predstavljajo osnovo teorije sferičnih t -načrtov, bomo dokazali več karakterizacij sferičnih t -načrtov. V nadaljevanju nas bo zanimala predvsem povezava med sferičnimi t -načrti in strukturami iz algebraične kombinatorike. Pokazali bomo, kako lahko sferične t -načrte skonstruiramo iz razdaljno-regularnih grafov in kakšno vlogo pri tem igrajo Kreinovi parametri asociativnih shem. Dokazali bomo tudi mejo linearnega programiranja, ki velja za najbolj temeljno neenakost v teoriji sferičnih t -načrtov. Delo bomo zaključili z opisom Godsilove posplošitve sferičnih t -načrtov.

In this thesis we deal with spherical t -designs in a real Euclidean space. By using properties of harmonic and homogeneous polynomials, which represent the foundation for the theory of spherical t -designs, we derive several characterizations of spherical t -designs. Afterwards we are mainly interested in the connection between spherical t -designs and structures from algebraic combinatorics. We show how it is possible to construct spherical t -designs from distance-regular graphs and what is the role of Krein parameters of associative schemes in this process. We also prove the linear programming bound, which is considered to be one of the most fundamental inequalities in the theory of spherical t -designs. We conclude the thesis by describing Godsil's generalization of spherical t -designs.

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Ključne besede: sferični t -načrti, asociativne sheme, razdaljno-regularni grafi, Kreinovi parametri, meja linearnega programiranja, polinomski prostori, harmonični polinomi, Gegenbauerjevi polinomi

Keywords: spherical t -designs, association schemes, distance-regular graphs, Krein parameters, linear programming bound, polynomial spaces, harmonic polynomials, Gegenbauer polynomials.

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