Povzetek

Delo obravnava Hartman-Wintnerjev zakon dvakratnega logaritma in vse, kar je potrebno za njegov dokaz preko Skorohodove vložitve.

V prvem poglavju predstavimo sam zakon dvakratnega logaritma, nekaj posledic in njegov obrat. Uvedemo potrebne oznake in razložimo, kako je delo sestavljeno in kje so bili uporabljeni posamezni viri.

V drugem poglavju navedemo znane izreke iz teorije verjetnosti in teorije mere, na katere se bomo sklicevali v nadaljevanju; večino tudi dokažemo. Nato pojasnimo pomen in pomembnost zakona dvakratnega logaritma.

V tretjem poglavju podamo relativno elementarna dokaza osrednjega izreka v najpomembnejših posebnih primerih – za normalne in za binomske slučajne spremenljivke.

V četrtem poglavju predstavimo Wienerjev proces in obravnavamo tiste njegove lastnosti, ki nam bodo prišle prav v nadaljevanju. To sta predvsem krepka lastnost Markova in princip zrcaljenja za Wienerjev proces.

V petem poglavju se spomnimo pogojnega matematičnega upanja, definiramo martingale in dokažemo pomemben izrek o opcijskem ustavljanju. Navedemo tudi nekaj drugih izrekov o martingalih, ki jih bomo potrebovali v naslednjem poglavju.

V šestem poglavju s pomočjo izrekov o Wienerjevem procesu in martingalih dokažemo dva od kjučnih rezultatov tega dela, izreka o Skorohodovi vložitvi. Nato dokažemo zakon dvakratnega logaritma za Wienerjev proces.

V zadnjem poglavju uporabimo rezultate iz prejšnjega poglavja, da končno dokažemo osrednji izrek v vsej splošnosti in njegov obrat. Za konec demonstriramo moč Skorohodove vložitve tako, da dokažemo še centralni limitni izrek.

Abstract

The thesis deals with the Hartman-Wintner law of the iterated logarithm and everything that it needed for its proof via the Skorokhod embedding.

In the first chapter the law of the iterated logarithm, some of its consequences and its converse are presented. The necessary notation is introduced and it is explained how the thesis is structured and where certain sources were used.

In the second chapter the theorems of probability and measure theory that will be referred to later in the thesis are stated; most of them are also proven. Then the meaning and the significance of the law of the iterated logarithm are explained.

In the third chapter there are rather elementary proofs of the law of the iterated logarithm in two most important special cases – when the distribution of random variables is normal (Gaussian) and when it is binomial.

In the fourth chapter the Wiener process is introduced. Those of its properties that will be useful in the following chapters are treated. These are especially the strong Markov property and the reflection principle.

In the fifth chapter the most important characteristics of conditional expectation are outlined and martingales are defined. Also, the optional stopping theorem is proven and some other theorems about martingales, which will be used later, are stated.

In the sixth chapter two key results, the Skorokhod embbeding theorems, are proven. To do this the theorems from the previous two chapters are used. Then the law of the iteratted logarithm for the Wiener process is proven.

In the last chapter the law of the iterated logarithm in its general form and the converse are finally proven, using crucially the results from the previous chapter. To conclude, the power of the Skorokhod embedding is demonstrated by proving the central limit theorem.

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