

# Povzetek

To diplomsko delo obravnava konveksne podgrafe v grafovskih produktih. V prvem poglavju tega dela definiramo osnovne pojme teorije grafov (razdelek 1.1). V razdelku 1.2 obravnavamo nekatere posebne družine grafov (kot so npr. drevesa, dvodelni grafi, ravninski grafi, ...). Razdelek 1.3 je namenjen homomorfizmom v grafih. Še posebej pomemben je razdelek 1.4, ki obravnava temeljne pojme metrične teorije grafov (izometričen podgraf, interval in konveksen podgraf). V razdelku 1.5 predstavimo  $k$ -konveksne podgrafe. Dokažemo tudi nekaj trditev (kot je npr. lema 1.55), ki odigrajo pomembno vlogo v nadaljevanju dela.

V drugem poglavju se seznanimo s štirimi osnovnimi grafovskimi produkti. Vsakemu produktu je namenjen svoj razdelek. Poleg osnovnih definicij spoznamo tudi nekatere njihove lastnosti. Za nadaljevanje sta najpomembnejša izreka 2.15 in 2.37.

Tretje poglavje je osrednje poglavje tega dela. Najprej spoznamo izrek 3.5, ki karakterizira konveksne podgrafe v kartezičnih produktih. Nato z uporabo te karakterizacije in vložitve v tri drevesa karakteriziramo še konveksne podgrafe v heksagonalnih grafih. V naslednjem razdelku je predstavljena domneva, ki karakterizira konveksne podgrafe v krepkih produktih. Zadnji razdelek pa je namenjen obravnavi konveksnosti v leksikografskih produktih.

**Math. Subj. Class. (MSC 2010):** 05C12, 05C76

**Ključne besede:**

izometričen podgraf, interval, konveksen podgraf, kartezični produkt, krepki produkt, direktni produkt, leksikografski produkt, heksagonalni graf, 2-konveksni podgraf

# Abstract

This thesis deals with convex subgraphs in product graphs. First chapter is an introduction to graph theory (section 1.1). Section 1.2 treats some special classes of graphs (such as trees, bipartite graphs, planar graphs, ...). Section 1.3 is devoted to graph homomorphisms. Section 1.4 is of the greatest importance since it investigates the most fundamental concepts (isometric subgraph, interval, and convex subgraph). Section 1.5 introduces the idea of a  $k$ -convex subgraph. Proofs of certain propositions (such as lemma 1.55) which play an important role in the following chapters are also given.

Chapter 2 introduces the four fundamental graph products. A separate section is devoted to each of them. Along with definitions, some nontrivial properties are also examined. The two theorems of the greatest importance for the following chapter are 2.15 and 2.37.

Chapter 3 is the primary chapter of this thesis. We first explain theorem 3.5, which characterizes convex subgraphs in Cartesian products. Then we show how the isometric embedding into three trees can be used to characterize convex subgraphs of hexagonal systems. The next section presents a conjecture, which classifies convex subgraphs of strong products. The final section is devoted to convexity in lexicographic products.

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**Keywords:**

isometric subgraph, interval, convex subgraph, Cartesian product, strong product, direct product, lexicographic product, hexagonal system, 2-convex subgraph

# Literatura

- [1] W. Imrich in S. Klavžar, *Product Graphs: Structure and Recognition*, John Wiley & Sons, New York, 2000.
- [2] R. H. Hammack, W. Imrich in S. Klavžar, *Handbook of Product Graphs (2nd Edition)*, CRC Press, Boca Raton (Florida), 2011.
- [3] I. Peterin, *Intervals and convex sets in strong product of graphs*, poslano v objavo.
- [4] B. S. Anand, M. Changat, S. Klavžar in I. Peterin, *Convex Sets in Lexicographic Products of Graphs*, Graphs Combin., v tisku.
- [5] J. Vrabec, *Metrični prostori*, DMFA Slovenije, Ljubljana, 1993.
- [6] T. H. Cormen, C. E. Leiserson, R. L. Rivest in C. Stein, *Introduction to Algorithms (2nd Edition)*, The MIT Press, Cambridge (Massachusetts), 2005.
- [7] B. Mohar in C. Thomassen, *Graphs on Surfaces*, The Johns Hopkins University Press, London, 2001.
- [8] T. Pisanski, B. Zmazek in J. Žerovnik, *An algorithm for  $k$ -convex closure and an application*, Int. J. Comput. Math. **78** (2001), 1–11.
- [9] T. Tantau, *The TikZ and PGF Packages: Manual for version 2.00*, dostopno na <http://ctan.ijs.si/tex-archive/graphics/pgf/base/doc/generic/pgf/pgfmanual.pdf>.