

## Povzetek

V diplomskem delu bomo spoznali vztrajno homologijo, ki je trenutno precej aktualno področje matematike, razvita pa je bila predvsem z namenom, da homologija, ki je zelo bogata in obsežna matematična teorija, postane nekoliko bolj aplikativno uporabna. Kljub tej osnovni motivaciji je področje precej zanimivo tudi s teoretičnega vidika.

Podrobno si bomo ogledali izrek o stabilnosti vztrajnostnih diagramov članka [4], ki je teoretično zagotovilo o aplikativni uporabnosti vztrajne homologije. Nato se bomo osredotočili na eno izmed osnovnih definicij, ki se je pred kratkim izkazala za problematično. Predstavili bomo zbirko protiprimerov, ki so plod lastnega raziskovalnega dela ([16]) in pokažejo, da pri originalni formulaciji omenjene definicije ena izmed osnovnih lem članka [4] odpove.

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**Ključne besede:** vztrajna homologija, stabilnost, homološka kritična vrednost, gladka mnogoterost, protiprimer

**Keywords:** persistent homology, stability, homological critical value, smooth manifold, counterexample

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