

Povzetek

Abstract

V delu obravnavamo problem določanja pasovnosti enostavnih, neuteženih grafov. Pasovnost grafa najprej posplošimo na p -pasovnost, $1 \leq p \leq \infty$. Običajno pasovnost potem nadomestimo z ∞ -pasovnostjo. Predstavljene so pomembnejše splošne ocene za ∞ -pasovnost, ki vsebujejo običajne invariante grafov. Nekatere od njih so nato še dodatno analizirane in prilagojene ostalim p -pasovnostim. Omenjene so tudi povezave med različnimi pasovnostmi. V nadaljevanju je vpeljana Laplaceova matrika grafa. Opravljena je obsežna analiza lastnosti te matrike in njenega spektra. Dokazane so nekatere zanimive povezave med običajnimi lastnostmi grafov in med drugo najmanjšo lastno vrednostjo λ_2 Laplaceove matrike grafa. Iz teh relacij so nato izpeljane spektralne ocene za p -pasovnosti, $p \in \{1, 2, \infty\}$, grafa. Omenjena je tudi prezerna pasovnost in njena povezava z Laplaceovim spektrom. V drugem delu je predstavljen hevristični algoritem za iskanje približka k za p -pasovnost optimalni označitvi grafa. Algoritem uporablja k drugi najmanjši Laplaceovi lastni vrednosti λ_2 pripadajoči lastni vektor $x^{(2)}$ in ima polinomsko časovno zahtevnost. Tako je to prvi praktično uporabni približni algoritem za ugotavljanje pasovnosti grafov, ki ga lahko uporabimo pri reševanju problemov, ki zahtevajo določanje pasovnosti večjih grafov, na primer pri izdelavi VLSI vezij. Čeprav je njegovo obnašanje v nekaterih izjemnih primerih slabo, pa na splošno daje zadovoljive rezultate. Pri kraji analizi napake algoritma so uporabljene tudi že izpeljane spektralne ocene. Na koncu je prikazana še uporaba teh ocen pri študiju pasovnosti slučajnih grafov.

The paper deals with the problem of determining the labeling of simple, unweighted graphs. We first generalize the concept of labeling to p -labeling, where $1 \leq p \leq \infty$. Then we replace the ordinary labeling by ∞ -labeling. Some general bounds for ∞ -labeling are given, which include the usual graph invariants. Some of them are further analyzed and adapted to other p -labelings. The Laplace matrix of a graph is introduced. A detailed analysis of its properties and spectrum is performed. It is shown that there are close connections between the usual graph properties and the second smallest eigenvalue of the Laplace matrix. From these relations spectral estimates for p -labelings, $p \in \{1, 2, \infty\}$, are derived. It is also shown that there is a connection between the ∞ -labeling and the spectrum of the Laplace matrix. In the second part of the paper a heuristic algorithm for finding an approximation to the p -labeling is presented. The algorithm uses the second smallest eigenvector of the Laplace matrix. Its computational complexity is polynomial. It is the first practical algorithm for determining the labeling of graphs, which can be used for solving problems that require determining the labeling of large graphs, for example, in VLSI design. Although its behavior in some special cases is poor, it generally gives good results. At the end the errors of the algorithm are analyzed using the spectral estimates. Finally, the application of these estimates in the study of random graphs is demonstrated.

Uporabljena literatura: S.Johnson, R.L.Stockmeyer, Some simplified NP-complete problems

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