Povzetek

V uvodnem poglavju so vpeljani nekateri osnovni pojmi teorije grafov. Dokazani so nekateri znani rezultati, na primer *Petersenov* izrek in *Tutteov* izrek o obstoju 1-faktorja v grafu. Drugo poglavje zajema učinkovit hevristični algoritem za iskanje Hamiltonovih ciklov v kubičnih grafih. Predstavljena je tudi uspešna procedura za iskanje Hamiltonovih ciklov v kubičnih grafih, ki deluje na principu sestopanja. V tretjem poglavju so podane tri predstavitve povezanih 1-tranzitivnih kubičnih grafov, ki so zbrani v knjigi *The Foster Census of Connected Symmetric Trivalent Graphs.* V nekaterih grafih iz te zbirke je opisani hevristični algoritem našel Hamiltonove cikle, čeprav doslej ni bilo znano, da so hamiltonski. V zadnjem poglavju si ogledamo grupo $PSL_2(7)$. Z opisanim hevrističnim algoritmom in proceduro sestopanja je dokazana hamiltonost vseh neizomorfnih kubičnih Cayleyevih grafov grupe $PSL_2(7)$.

Ključne besede

Cayley graf, kubičen graf, Hamiltonov cikel, projektivna specialna linearna grupa.

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Abstract

Basic definitions from graph theory are considered. Some well known results as *Petersen's* theorem and *Tutte's* theorem about the existence of 1-factor in a graph are proved. A successful heuristic algorithm for finding Hamilton cycles in cubic graphs is described. Also an efficient backtracking procedure for computing Hamilton cycles is presented. Three representations of connected 1-transitive cubic graphs from *The Foster Census of Connected Symmetric Trivalent Graphs* are described. In several of them the heuristic algorithm found Hamilton cycles although it was not known before whether they are hamiltonian or they are not. The group $PSL_2(7)$ is discussed and all nonisomorphic cubic Cayley graphs of group $PSL_2(7)$ are shown to be hamiltonian.

Key words

Cayley graph, cubic graph, Hamilton cycle, projective special linear group.

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