Povzetek

Opisane so nekatere lastnosti, ki veljajo v kolobarjih polinomov. Izpeljana je zgornja meja za velikost determinante matrike, ki ima za člene polinome več spremenljivk. Podane so ocene za časovne zahtevnosti in opisani manj očitni algoritmi za osnovne računske operacije v kolobarju celih števil, obsegu racionalnih števil in obsegu ostankov po praštevilskem modulu ter v kolobarju polinomov več spremenljivk s koeficienti iz prej omenjenih kolobarjev. V tretjem poglavju so vpeljani nekateri osnovni pojmi iz teorije grafov ter opisana znana algoritma: madžarska metoda za maksimalno prirejanje v dvodelnem grafu in Kuhn-Munkresov algoritem za optimalno prirejanje v uteženem dvodelnem grafu. Dodan je še algoritem za optimalno prirejanje dane moči. Izpeljanih je več algoritmov za računanje generičnega ranga matrike s členi iz kolobarja polinomov več spremenljivk: Gaussova eliminacija, Edmondsov algoritem in njegova modularna varianta ter algoritem z vstavljanjem vrednosti. Pri zadnjih dveh pride do izraza povezava s problemi prirejanj v dvodelnih grafih. Pri vseh omenjenih algoritmih je dokazana polinomskost prostorske in časovne zahtevnosti. Na koncu je omenjen primer uporabe izpeljanih algoritmov za ugotavljanje vodljivosti sistemov.

Ključne besede: algoritem, determinanta, dvodelen graf, generični rang, matrika, polinom, prirejanje, rang, strukturni rang, učinkovitost, vodljivost sistemov.

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Abstract

Some properties of polynomial rings are described. An upper bound for the size of the determinant of a matrix with entries being polynomials of several variables is computed. Upper bounds for the time complexity are given and some non-trivial algorithms are described for performing basic computations in the ring of integers, in the field of rationals, in the field of integers modulo prime number, and in the ring of polynomials of several variables with coefficients from one of the rings mentioned above. In the third chapter some basic definitions from graph theory are considered and two well-known algorithms, i.e. the Hungarian method for maximum matching in a bipartite graph and the Kuhn-Munkres algorithm for optimal matching in a weighted bipartite graph, are described. Also, an algorithm for optimal matching of a given order is added. Several algorithms for computing the generic rank of a matrix whose entries are polynomials of several variables are presented; i.e. the Gaussian elimination, the Edmonds' algorithm with its modular version, and the interpolating algorithm. In the latter two algorithms a relation to problems regarding matchings in bipartite graphs is shown. Polynomial time and space complexity of all mentioned algorithms is proved. At the end systems controllability is mentioned as an example of application of the described algorithms.

Key words: algorithm, bipartite graph, complexity, determinant, generic rank, matching, matrix, polynomial, rank, systems controllability, term rank.

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