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Povzetek

Naloga obravnava risanje celično vloženih grafov v orientabilne ploskve. Ploskve predstavimo s poligoni v ravnini, ki imajo označene in usmerjene stranice, in rišemo grafe v te poligone. Proučujemo predvsem take risbe, kjer je rob poligona sestavljen iz povezav grafa ali pa so vse točke grafa v notranjosti poligona in nobena povezava ne gre čez rob poligona več kot enkrat. Najdemo potreben in zadosten pogoj, ki mu mora zadoščati celično vložen graf, da ga lahko na tak način narišemo v standardni poligon ploskve. Izpeljemo tudi polinomski algoritem, ki ugotovi, ali se graf da tako narisati. V primeru, da je to mogoče, graf tudi nariše v standardni poligon ploskve.

Abstract

In this work some possibilities for drawing graphs which are 2-cell embedded in orientable surfaces are considered. We represent surfaces by polygons in the plane, with their sides oriented and labelled, and then draw graphs in these polygons. We are interested in drawings where the boundary of the polygon consists of the edges of the graph as well as in drawings in which all the vertices of the graph are inside the polygon and no edges cross the boundary of the polygon more than once. A necessary and sufficient condition is found for a given 2-cell embedded graph to be drawn in the standard polygon of the surface in such a way. A polynomial algorithm is presented which determines whether or not a given 2-cell embedded graph can be drawn in the standard polygon of the surface in that way. In the affirmative, it also produces such a drawing of the graph.

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