

## POVZETEK

V tem delu iščemo analitične rešitve inverznega kinematičnega problema za neredundantne robotske manipulatorje opisane z Denavit-Hartenbergovimi parametri. Matematičen opis problema nam da sistem polinomskih enačb. Za ideal, ki pripada temu sistemu, poiščemo z Buchbergerjevim algoritmom Gröbnerjevo bazo – trikoten sistem. Ko ta sistem rešimo, dobimo natanko vse rešitve začetnega sistema. Poleg samega reševanja inverznega kinematičnega problema si z Gröbnerjevimi bazami lahko pomagamo pri iskanju optimalnih trikotnih sistemov za razrede robotskih manipulatorjev. Različne stopnje reševanja inverznega kinematičnega problema in analiza trikotnih sistemov so predstavljene s primeri v programskih paketih *Mathematica* in *Maple*.

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**Key words:** Gröbner bases, Buchberger algorithm, ideals, resultants, inverse kinematics, robotics, manipulators, specialized analysis

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