Povzetek

Opazovali bomo problem pravilnega k-barvanja grafa reda n z maksimalno stopnjo Δ. Konstruirali bomo enostavni markovski verigi, ki bosta ergodični in bosta konvergirali k svoji stacionarni porazdelitvi v polinomskem času. Pokazali bomo, da dinamika zamen konvergira k stacionarni porazdelitvi v času $O(nk \log n)$, ko je $k > \frac{11}{6}\Delta$, in da glauberjeva dinamika na istem območju konvergira k stacionarni porazdelitvi v času $O(n^2 k \log n \log k)$.

Math. Subj. Class (2002): 05C15, 68W40, 68W20, 90C35

Ključne besede: graf, pravilno barvanje, markovska veriga, algoritem WSK, glauberjeva dinamika, dinamika zamen, mešalni čas, hitro mešanje, Pottsov model.

Abstract

We study the problem of proper k-colorings of a graph of order n and maximum degree Δ. The design of two simple Markov chains, which are ergodic and converge to their stationary distributions in polynomial time is presented. If $k > \frac{11}{6}\Delta$ both Markov chains are rapidly mixing, flip dynamics with mixing time $O(nk \log n)$ and Glauber dynamics with mixing time $O(n^2 k \log n \log k)$.

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Key words: graph, proper coloring, Markov chain, WSK algorithm, Glauber dynamics, flip dynamics, mixing time, rapidly mixing, Potts model.

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