# Kratek povzetek vsebine

Raziskujemo metodo kolokacije pri reševanju robnih problemov sistema linearnih diferencialno - algebraičnih enačb (DAE) z nekonstantnimi koeficienti, ki nima omejitev glede na indeks $\nu$. Sistem s pomočjo znanih metod pretvorimo na ekvivalenten sistem indeksa 1, ki sestoji iz $d$ diferencialnih ter $a = n - d$ algebraičnih enačb. Opisana je tako splošna teorija metode kolokacije, kot tudi osnovne lastnosti sistema DAE. Za izbiro kolokacijskih točk, ki so ničle ortogonalnih polinomov, uporabimo najprej Radaujevo shemo, nato pa še kombinacijo Gaussove (za diferencialni del) in Lobattove sheme (za algebraični del). Tako dobimo dve metodi, ki nam data zvezno, odsekoma polinomsko funkcijo $x_\pi$ stopnje $k$, ki je približna rešitev za točno rešitev $x$. Pod šibkimi pogoji nam obe metodi data enolično rešitev, ki jo izračunamo stabilno. Če je enolična rešitev $x$ dovolj gladka, dobimo konvergenco reda $\min\{k+1, 2k-1\}$ v prvem in reda $k+1$ v drugem primeru ter poleg tega v stičnih točkah še superkonvergenco reda $2k-1$ v prvem in reda $2k$ v drugem primeru. Prikazanih je nekaj numeričnih zgledov, ki predstavijo učinkovitost teh dveh metod.

# Abstract

Collocation methods for solving boundary value problem for linear differential - algebraic equations (DAE) with variable coeficients and no restriction on the index $\nu$ are outlined. A well-known regularization procedure yields an equivalent problem of index 1 which consists of $d$ differential and $a = n - d$ algebraic equations. General theory of collocation is studied, as well as some basic properties of the DAE. Collocation knots are chosen by the Radau schemes as well as by a combination of the Gauss schemes for the diferential part and of the Lobatto schemes for the algebraic part. Thisway two methods are derived that guarantee the existence of a continous piecewise polinomial $x_\pi$ of degree $k$, which is an approximation for the solution $x$. Under weak assumptions, we obtain unique and stable solvable solution. If the unique solution $x$ is sufficiently smooth, convergence of order $\min\{k + 1, 2k - 1\}$ in the first case and of order $k + 1$ in the second case is shown. Moreover, superconvrgence at mesh points of order $2k - 1$ in the first case and of order $2k$ in the second case is obtained. Several numerical examples are added to illustrate effectivenes of this two methods.

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