Povzetek

V prvem poglavju ponovimo nekaj osnovne teorije lastnih vrednosti. Opišemo Arnoldijevo in Lanczosevo metodo za kreiranje baze podprostora Krilova. Nazadnje si ogledamo Rayleigh-Ritzev algoritem, iterativno metodo, ki poišče najboljše približke lastnih vrednosti za zoženi problem.

Drugo poglavje obravnava metode podprostorov Krilova, iterativne metode za reševanje sistema linearnih enačb, ki uporabljajo tako imenovano "črno škatlo" - program, ki vrne produkt matrike z vektorjem. Z eno od metod rešimo Jacobi-Davidsonovo korekcijsko enačbo. Podrobneje si ogledamo metode GMRES, MINRES, CG, Bi-CG in Bi-CGSTAB. Na koncu poglavja je opisano še predhodno pogojevanje, ki pri velikih razpršenih sistemih izboljša lastnosti slabo pogojenega sistema.

V tretjem poglavju obravnavamo Jacobi-Davidsonovo metodo, njene različice za hermitske in nehermitske matrike oziroma za ekstremne in notranje lastne vrednosti. Pri hermitskih matrikah nam algoritem vrne lastne pare, pri nehermitskih pa delno Schurovo formo. Ekstremne lastne vrednosti računamo z navadnimi Ritzevimi vrednostmi, za notranje lastne vrednosti pa so zaradi neregularne konvergence primernejše harmonične Ritzeve vrednosti.

V zadnjem poglavju je navedenih nekaj numeričnih primerov.

Ključne besede:
lastne vrednosti in lastni vektorji, metode podprostorov Krilova, Jacobi-Davidsonova metoda, iterativne metode za reševanje problema lastnih vrednosti, harmonične Ritzeve vrednosti, Ritzeve vrednosti.

Abstract

In the first chapter we repeat some of the eigenvalue theory. We describe Arnoldi's and Lanczos's algorithm for building a basis for the Krylov subspace. We look inside of the Rayleigh-Ritz method, iterative method, which finds the best approximation for the eigenvalues of the projected problem. Those approximations are named Ritz values.

Second chapter is dedicated to Krylov subspace methods, iterative methods for solving large sparse systems of linear equations. We used them for solving Jacobi-Davidson correction equation. More detailed we look at GMRES, MINRES, Bi-CG and Bi-CGSTAB method. Solving of the large sparse systems is improved by preconditioning, which is described in the last part of chapter.

In the third chapter we completely describe Jacobi-Davidson method, all its variants for hermitian and non-hermitian matrices, exterior and interior eigenvalues. Jacobi-Davidson algorithm for hermitian problems returns eigenvalue pairs and for non-hermitian problems returns partial Schur form. We approximate exterior eigenvalues with Ritz values. When we are looking for interior eigenvalues, we have to replace Ritz values with harmonic Ritz values. The reason is iregular convergence.

Numerical examples are stated in the last chapter.

Key words:
eigenvalues and eigenvectors, Krylov subspace methods, Jacobi-Davidson method, iterative methods for solving eigenvalue problem, harmonic Ritz values, Ritz values.

Math. Subj. Class. (2000): 15A18, 49M15, 90C53, 93B60, 93C05

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