POVZETEK

Polinomska interpolacija v več spremenljivkah je eno pomembnih področij v numerični aproksimaciji. Je precej zahtevnejša kot v eni dimenziji, zato je še vedno deležna precejšne pozornosti in številnih raziskav. Poleg zgodovinskih izhodišč sem predstavil pregled pomembnejših sodobnih rezultatov s tega področja. Interpolacijski problem sem preučil z dveh zornih kotov. V prvem pristopu rešujemo problem, kako konstruirati interpolacijske točke, ki bodo določale korekten interpolacijski problem v danem interpolacijskem prostoru. V drugem pristopu so interpolacijske točke dane in iščemo interpolacijski prostor, ki porodi korektno nalogo. Posebej sem se osredotočil še na izražave in ocene napake ter nakazal vzporednice s področjem računalniške algebre.

ABSTRACT

The multivariate polynomial interpolation is one of the important subjects in the numerical approximation. It is much more complex than the univariate case and therefore it is still very much under active research. Beside the historical survey the main recent results on the multivariate polynomial interpolation there are presented. The interpolation problem is considered from two points of view. In the first approach the problem how to find the data points that imply the correct interpolation problem for the given interpolation space is studied. In the second approach, the interpolation points are given and an interpolation space which gives rise to the correct interpolation problem is searched for. Error formulas and error estimates are outlined too, and the relationship with the computer algebra is indicated.

Math. Subj. Class. (2000): 41A05, 41A10, 41A63, 65D05

Ključne besede: interpolacija, polinomi več spremenljivk, deljene diference, izražava napake, naravne mreže, mreže generirane s snopi hiperravnin, interpolacijski prostor minimalne stopnje

Key words: interpolation, multivariate polynomials, divided differences, error representation, natural lattices, pencils-lattices, minimal degree interpolation space

Literatura

[1] C. de Boor: A multivariate divided difference, *Approximation Theory VIII, Vol. 1: Approximation and Interpolation,* C. K. Chui and L. L. Schumaker, (ed.), World Scientific Publishing Co., 1995, 87 – 96.

[2] C. de Boor: The error in polynomial tensor-product, and in Chung-Yao, interpolation, *Surface Fitting and Multiresolution Methods,* A. Le Méhauté, C. Rabut and L. L. Schumaker, (ed.), Vanderbilt University Press, 1997, 35 – 50.

[3] C. de Boor and A. Ron: On multivariate polynomial interpolation, *Constr. Approx.,* 6 (1990), 287– –302.

[4] K. C. Chung and T. H. Yao: On lattices admitting unique Lagrange interpolation, *SIAM J. Num. Anal.,* 14 (1977), 735 – 743.

[5] P. G. Ciarlet and C. Wagschal: Multipoint Taylor formulas and applications to the finite element method, *Num. Math.,* 17 (1971), 84 – 100.

[6] M. Gasca: Multivariate polynomial interpolation, *Computation of Curves and Surfaces*, W. Dahmen, M. Gasca and C. A. Micchelli, (ed.), Kluver Academic Publishers, 1990, 215 – 236.

[7] M. Gasca and E. Lebrón: On Aitken-Neville formulae for multivariate interpolation, *Numerical approximation of partial differential equations*, E. L. Ortiz, (ed.), North Holland, 1987, 133 – 140.

[8] M. Gasca and J. I. Maeztu: On Lagrange and Hermite interpolation in $\mathbb{R}^k$, *Numer. Math.,* 39 (1982), 1 – 14.

[9] M. Gasca and T. Sauer: On the history of multivariate polynomial interpolation, preprint.

[10] M. Gasca and T. Sauer: Polynomial interpolation in several variables, *Advances in Computational Mathematics*, v tisku.

[11] M. Gasca and T. Sauer: On bivariate Hermite interpolation with minimal degree polynomials, *SIAM J. Numer. Anal.,* Vol. 37, No. 3, (1999), 772 – 798

[12] E. Isaacson and H. B. Keller: *Analysis of Numerical Methods,* John Wiley & Sons, New York 1966.

[13] J. Kozak: *Numerična in računalniška matematika: Aproksimacija v več dimenzijah,* 4. *del*, Ljubljana 1985.

[14] K. S. Kunz: *Numerical Analysis,* McGraw-Hill Book Company, 1957.

[15] S. L. Lee and G. M. Phillips: Construction of lattices for Lagrange interpolation in Projective space, *Constructive Approximation,* 1991, 283 – 297.

[16] R. A. Lorentz: *Multivariate Birkhoff Interpolation,* Number 1516 in Lecture Notes in Mathematics, Springer Verlag, 1992.

[17] C. A. Micchelli: A constructive approach to Kergin interpolation in $\mathbb{R}^k$: multivariate B-splines and Lagrange interpolation, *Rocky Mountain J. Math.,* 10 (1979), 485 – 497.

[18] C. A. Micchelli and P. Milman: A formula for Kergin interpolation in $\mathbb{R}^k$, *J. Approx. Th.,* 29 (1980), 294 – 296.

[19] R. A. Nicolaides: On a class of finite elements generated by Lagrange interpolation, *J. Numer. Anal.,* 9 (1972), 435 – 445.

[20] T. Sauer: Polynomial interpolation of minimal degree, *Numer. Math.,* 78 (1997), 59 – 85.

[21] T. Sauer: Polynomial interpolation of minimal degree and Gröbner bases, *Groebner Bases and Applications (Proc. of the Conf. 33 Years of Groebner Bases), volume* 251 *of London Math. Soc. Lecture Notes*, B. Buchberger and F. Winkler, (ed.), Cambridge University Press (1998), 483 – 494.

[22] T. Sauer and Yuan Xu: On multivariate Lagrange interpolation, *Math. Comp.,* 64 (1995), 1147 – –1170.

[23] H. C. Thacher Jr. and W. E. Milne: Interpolation in several variables, *J. SIAM,* 8 (1960), 33 – 42.