

# Povzetek

Diplomsko delo je sestavljeno iz dveh delov. V prvem delu najprej obravnavamo vprašanje obstoja dekompozicije poljubnega polnega grafa ali polnega multigrafa na izbran polni podgraf s pomočjo uravnoveženih nepopolnih bločnih načrtov, ki predstavljajo prav te dekompozicije. Pri tem uporabljamo končne afine in projektivne geometrije ter Boseove direktne in Hananijeve rekurzivne konstrukcijske metode. Na koncu prvega dela podamo še Wilsonov izrek, ki nam da asimptotično rešitev vprašanja obstoja dekompozicije poljubnega polnega usmerjenega grafa na izbran (ne nujno poln) usmerjen podgraf. Izrek dokažemo s pomočjo paroma uravnoveženih načrtov in nekaterih drugih Wilsonovih dognanj.

V drugem delu podamo asimptotično rešitev problema pakiranja poljubnega polnega grafa z izbranim podgrafom, ki sta jo dokazala Caro in Yuster. V dokazu potrebujemo Erdős-Gallaiev izrek, ki ga dokažemo s pomočjo Tutteove teorije alternirajočih sprehodov.

Math. Subj. Class. (2000): 05C70, 05B05, 05C07.

Ključne besede: dekompozicija grafa, bločni načrt, pakiranje, zaporedje stopenj točk.

Keywords: graph decomposition, block design, packing, degree sequence.

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