Povzetek

V delu so podrobneje predstavljeni homomorfizmi grafov in uporaba le-teh pri barvanju grafov ter osnovne lastnosti krožnega kromatičnega števila grafa. Obdelani so tudi krožni pretoki in njihova povezava s krožnim barvanjem, predstavljena pa so tudi nekatera druga grafovska barvanja. Ključni del diplomskega dela predstavlja izrek, ki pravi, da ima ravninski graf z notranjim obsegom vsaj $\frac{20t-2}{3}$ krožno kromatično število kvečjemu $2+\frac{1}{t}$, kar je bolje od dosedanjih rezultatov. Dobljeni rezultat sledi iz splošnejšega dognanja, v katerem grafe homomorfno preslikujemo v posebne slike, pri čemer imamo dan notranji obseg grafa in največjo povprečno stopnjo vozlišč. V ostalih primerih uporabe se dotaknemo še deljenega, orientiranega in acikličnega barvanja ter homomorfizmov v mešane grafe z obarvanimi povezavami.

Abstract

This paper shows in detail how graph homomorphisms can be used to present the colouring of graphs. It also gives a close look at circular chromatic number of a graph and its basic properties. Circular flows and their connection with circular colouring are also mentioned, as well as some of the other variants of colouring. The main part of the paper represents a theorem which says that a planar graph with girth at least $\frac{20t-2}{3}$ has circular chromatic number at most $2+\frac{1}{t}$, improving earlier result. This follows from a general result establishing homomorphisms into special targets for graphs with given girth and given maximum average degree. Other applications concern fractional, oriented and acyclic colouring and homomorphisms into mixed graphs with colored edges.

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Ključne besede: barvanje grafa, homomorfizem grafa, ravninski graf, krožno barvanje, orientirano barvanje, prenašanje naboja, t-lepi grafi

Keywords: graph coloring, graph homomorphism, planar graph, circular coloring, oriented coloring, discharging, t-nice graphs

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