Povzetek

Motivacija za to diplomsko delo je Pickov izrek, ki pravi, da obstaja omejena holomorfna funkcija na enotskem disku s supremum normo največ ena, ki interpolira vrednosti w_i v λ_i, natanko tedaj, ko je matrika

$$\left[\frac{1-w_i\overline{w_j}}{1-\lambda_i\overline{\lambda}_j}\right]_{i,j=1}^N$$

pozitivno semi–definitna. Ker je prostor omejenih holomorfnih funkcij na enotskem disku ($H^\infty(\Delta)$) ravno prostor multiplikatorjev za Hardyev prostor, posplošimo Pickov problem na interpolacijo z multiplikatorji splošnih Hilbertovih funkcijskih prostorov. Po bijektivni korespondenci med Hilbertovimi funkcijskimi prostori in jedri je jedro za Hardyev prostor Szegővo jedro $k^s(\lambda_i,\lambda_j) = (1-\lambda_i\overline{\lambda}_j)^{-1}$. Postavimo vprašanje, kakšni so ti Hilbertovi funkcijski prostori, da je pozitivna semi–definitnost matrike $\left[(1-w_iw_j^*)k(\lambda_i\lambda_j)\right]_{i,j=1}^N$ potreben in zadosten pogoj za obstoj multipikatorja z normo največ ena ter z interpolacijskimi lastnostmi. Ker bi z multiplikatorji radi zajeli tudi take z matričnimi vrednostmi, vprašanje posplošimo na tenzorski produkt Hilbertovega funkcijskega prostora in končno razsežnega Hilbertovega prostora. Prvo "preprosto" in "uporabno" karakterizacijo takih Hilbertovih funkcijskih prostorov je podal P.Quiggin (1993). V nadaljevanju se zopet osredotočimo na posplošen Pick–Nevanlinnin problem za Hardyeve prostore. Pokažemo, kdaj ima dan Pickov problem natanko eno rešitev in kakšna je ta rešitev. Če pa ima več rešitev, pa podamo parametrizacijo vseh rešitev.

Abstract

Pick theorem is taken as a basic motivation for this thesis. By the theorem, there is holomorphic function, bounded by one, on unit disc such that maps each point λ_i to the corresponding value w_i if and only if the matrix

$$\left[\frac{1-w_i\overline{w_j}}{1-\lambda_i\overline{\lambda}_j}\right]_{i,j=1}^N$$

is positive semi–definite. Since the space of bounded holomorphic functions ($H^\infty(\Delta)$) on the unit disk is the multiplier space of the Hardy space, Pick problem can be generalized to the interpolation with multipiers of a general Hilbert function space. By the bijective correspondence between Hilbert function spaces and kernels, we get Szegő kernel $k^s(\lambda_i,\lambda_j) = (1-\lambda_i\overline{\lambda}_j)^{-1}$ as the kernel for Hardy space. The question is posed, which Hilbert function spaces have the property that a multipier of norm less then or equal to one satisfies interpolation conditions if and only if the matrix $\left[(1-w_iw_j^*)k(\lambda_i\lambda_j)\right]_{i,j=1}^N$ is positive semi-definite. Since we are also interested in the matrix valued multipliers, the same question is asked for a tensor product of the Hilbert function space and some finite dimensional Hilbert space. The first "elementary" and "usable" answer to this question was given by P.Quigin (1993). Lastly, we return to the generalized Pick–Nevanlinna problem for Hardy space. Conditions are presented when a given Pick problem has only one solution and that solution is presented. If the problem has more than one solution, a complete parametrization of solutions is given.

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Ključne besede: Pickova interpolacija, prostori analitičnih funkcij, Hardyev prostor, Hilbertov funkcijski prostor, teorija operatorjev, jedro, tenzorski produkt Hilbertovih prostorov, Nevanlinna–Pickov problem, realizacijska formula.

Keywords: Pick interpolation, spaces of analytic functions, Hardy space, Hilbert function space, operator theory, kernel, Hilbert space tensor product, Nevanlinna–Pick problem, realization formula.

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