Povzetek

V diplomskem delu bomo predstavili sistem mas in vzmeti, ki je podan z matrikama M in K. Zanimalo nas bo, kako poiskati izospektralne sisteme začetnemu sistemu. Kako to storimo, je odvisno od tega, kakšne oblike je matrika M.

Če je matrika M diagonalna, lahko poiščemo izospektralne sisteme na štiri načine: z zamenjavo mas in vzmeti za sistem brez desnega nosilca, z uporabo nedoločnosti redukcije na posplošeno obliko, z uporabo enega ali več korakov LL^{T}-algoritma ali z uporabo enega ali več korakov QR-algoritma. Pokazali bomo, da lahko v tem primeru pridemo iz poljubnega začetnega sistema do poljubnega izospektralnega končnega sistema.

V primeru, ko je matrika M tridiagonalna, si bomo ogledali, kako rekonstruiramo matriki K in M in kako poiščemo družino izospektralnih sistemov. Zanimalo nas bo tudi, kako najti družino sistemov, ki imajo določen odziv lastne funkcije na prostem koncu.

Na koncu si bomo ogledali numerične primere predstavljenih algoritmov in poskušali poiskati izospektralne sisteme z določenimi lastnosti, kot je npr. enakost diagonalnih ali obdiagonalnih mas matrike M.

Abstract

The work concerns systems of masses and ideal massless springs, given with matrices M and K. We are interested in finding isospectral systems to a given one. The construction depends on the form of matrix M.

If M is diagonal, four ways are given for constructing an isospectral system: by using interchange for a cantilever ($k_{n+1} = 0$), by using indeterminacy associated with the reduction to standard form, by using one or more shifted LL^{T} factorizations and reversals or by using one or more shofted QR factorizations and reversals. It is shown, when M is diagonal, that we can pass from any system to any other isospectral system.

If M is tridiagonal, we will show, how the overall mass and stiffness matrices M and K can be reconstructed from the matrix A, and how we find a family of isospectral systems. We are also interested in finding a family of isospectral systems, which has a defined response to a unit load at the free end.

At the end we will present some numerical examples of introduced algorithms and we will try to find isospectral systems with some properties, such as equality of diagonal or subdiagonal masses of M.

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Literatura

[1] M. T. Chu, G. H. Golub: *Inverse Eigenvalue Problems: Theory, Algorithms, and Applications.* Oxford University Press, Oxford, 2005.

[2] G. M. L. Gladwell: *Inverse Problems in Vibration.* Kluwer academic publishers, Dordrecht, 1986.

[3] G. M. L. Gladwell: *On isospectral spring-mass systems.* Inverse problems 11, 1995, str. 591-602.

[4] G. M. L. Gladwell: *Inverse vibration problems for finite-element models.* Inverse problems 3, 1997, str. 311-322.

[5] G. M. L. Gladwell: *Total positivity and the QR algorithm.* Linear algebra and it's applications 271, 1998, str. 257-272.

[6] G. M. L. Gladwell: *Inverse finite element vibration problems.* Journal of sound and vibration 211, 1999, str. 309-324.