Povzetek

V diplomi se ukvarjamo s simetričnim nenegativnim inverznim problemom lastnih vrednosti, kjer nas zanima kakšni so potrebni in zadostni pogoji za seznam realnih števil, da obstaja nenegativna simetrična matrika, ki ima ta seznam za spekter. Ta problem spada v razred nenegativnih inverznih problemov lastnih vrednosti, ki je eden najzanimivejših problemov v teoriji nenegativnih matrik. Kljub intenzivnem študiju, pa je ta razred ostal nerešen za sezname, ki imajo več kot štiri elemente.

V prvem poglavju predstavimo inverzni problem lastnih vrednosti. Podamo njegovo klasifikacijo in opišemo glavne tri razrede: strukturiran, delno opisan in parametričen inverzni problem lastnih vrednosti.

V drugem poglavju govorimo o spektralnih lastnostih nenegativnih matrik. Predstavimo Perron-Frobeniusov izrek, ki igra pomembno vlogo v teoriji nenegativnih matrik in Cauchyev izrek o prepletanju. Na koncu poglavja povemo zvezo med spektrom in diagonalnimi elementi simetrične nenegativne matrike.

V tretjem poglavju predstavimo potrebne in zadostne pogoje za nenegativni in simetrično nenegativni problem lastnih vrednosti. Povemo nekaj o združevanju že predstavljivih seznamov in o operacijah, ki jih ohranjajo.

V četrtem poglavju se posvetimo simetričnemu nenegativnemu inverznemu problemu lastnih vrednosti, kjer imamo poleg spektra dane še diagonalne elemente. Veliko simetrično predstavljivih seznamov lahko dobimo z združevanjem že znanih simetrično predstavljivih seznamov.

Delo zaključimo z uporabo inverznega problema lastnih vrednosti.

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