R. Tomc. *Medialna os ravninskega območja.* Diplomsko delo, Univerza v Ljubljani, Fakulteta za matematiko in fiziko, Oddelek za matematiko, 2014.

POVZETEK

Medialna os ravninskega območja je definirana kot množica središč maksimalnih diskov, ki so vsebovani v območju. V diplomskem delu je dokazano, da je medialna os ravninskega območja, katerega rob je sestavljen iz končnega števila realno analitičnih krivulj, povezan ravninski graf s končnim številom vozlišč in povezav, pri čemer je vsaka povezava realno analitična krivulja.

V drugem poglavju je natančno opisan Voronoijev diagram. Opisana je tudi Delaunayjeva triangulacija in njena povezava z Voronoijevimi diagrami. Dokazano je, da je za dano končno množico $\mathcal{S}$ paroma različnih točk, za katero velja, da točke ležijo v splošni legi, ekvivalentno konstruirati Voronoijev diagram ali Delaunayjevo triangulacijo, saj lahko enega preprosto dobimo iz drugega. Natančneje je opisan en algoritem za izračun Delaunayjeve triangulacije.

Denimo, da izberemo dovolj gosto vzorčenje $\mathcal{S}$ na robu gladkega območja Ω in izračunamo Voronoijev diagram za $\mathcal{S}$. Pokazano je, da ima tedaj medialna os unije Voronoijevih diskov, ki imajo središča v Voronoijevih točkah znotraj Ω, zelo preprosto strukturo. Medialna os je enaka uniji vseh notranjih Voronoijevih točk in Voronoijevih robov, ki jih povezujejo. Na tak način smo gladko območje Ω aproksimirali z unijo notranjih Voronoijevih diskov in posledično smo tudi medialno os območja Ω aproksimirali z medialno osjo unije notranjih Voronoijevih diskov.

Ključne besede: medialna os,
transformacija medialne osi,
triangulacija,
Delaunayjeva triangulacija,
Voronoijev diagram.

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ABSTRACT

The medial axis of a planar domain is defined as a set of centers of maximal discs contained in a domain. In this work it is shown that the medial axis of a planar domain with a boundary that consists of a finite number of real analytic curves is a connected geometric graph with finitely many vertices and edges, where each edge is a real analytic curve.

The second chapter describes in detail a Voronoi diagram. Moreover, a Delaunay triangulation is presented and its connexion to a Voronoi diagram is revealed. It is proven that for a given finite set $\mathcal{S}$ of pairwise different points that lie in a general position it is equivalent to construct a Voronoi diagram or a Delaunay triangulation, since one can easily be obtained from the other. Furthermore, an algorithm for computing the Delaunay triangulation is described.

Suppose that a dense sampling $\mathcal{S}$ of a smooth boundary of a planar domain Ω is chosen and a Voronoi diagram for $\mathcal{S}$ is computed. It is shown that the medial axis of the union of Voronoi discs with centers in Voronoi vertices inside Ω has a very simple structure. It is the union of all Voronoi vertices inside Ω and the Voronoi edges connecting them. In this way a smooth domain Ω can be approximated by the union of inner Voronoi discs and consequently a medial axis of Ω is approximated with the medial axis of the union of inner Voronoi discs.

Keywords: medial axis,
medial axis transform,
triangulation,
Delaunay triangulation,
Voronoi diagram.

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Literatura

[1] J. Kepler. *Strena seu de nive sexangula.* Frankfurt, 1611.

[2] H. Blum. *A transformation for extracting new descriptors of shape.* V knjigi: W. W. Dunn, editor, *Proc. Symp. Models for the Perception of Speech and Visual Form*, pp. 362–380. MIT Press, Cambridge, MA, USA, 1967.

[3] S. G. Krantz, R. P. Harold. *Distance to C^k hypersurfaces.* J. of Differential Equations, 40, 116–120, 1981.

[4] T. M. Liebling, L. Pournin. *Voronoi Diagrams and Delaunay Triangulations: Ubiquitous Siamese Twins.* Documenta Mathematica, Extra Volume ISMP, 419–432, 2012.

[5] G. Voronoi. *Nouvelles applications des paramètres continus à la théorie des formes quadratiques.* J. Reine Angewandte Mathematik, 134, 198–287, 1908.

[6] J. J. Risler. *Mathematical Methods for CAD.* Cambridge University Press, Cambridge, 1992.

[7] H. Edelsbrunner, T. S. Tan. *A quadratic time algorithm for minmax length triangulation.* Proceedings of the 32nd IEEE Symposium of Foundations of Computer Science, pp. 414–423, 1991.

[8] C. Levcopoulos, A. Lingas. *Fast algorithms for greedy triangulations.* BIT, vol. 32, no. 2, pp. 280–296, 1992.

[9] H. Edelsbrunner, T. S. Tan, R. Waupotitsch. *An $\mathcal{O}(n^2\log(n))$ time algorithm for the minmax angle triangulations.* Proceedings of the 6th Annual Symposium on Computational Geometry, pp. 44–52, 1992.

[10] D. Eppstein. *Approximating the minimum weight triangulation.* Proceedings of the 3rd Annual ACM-SIAM Symposium on Discrete Algorithms, pp. 48–57, 1992.

[11] L. D. Floriani, E. Puppo. *An On-Line Algorithm for Constrained Delaunay Triangulation.* Graphical Models and Image Processing, vol. 54, no. 4, pp. 290–300, 1992.

[12] M. van Kreveld. *Digital elevation models and TIN algorithms.* V knjigi: M. van Kreveld, J. Nievergelt, T. Roos, P. Widmayer (Eds.). *Algorithmic Foundations of GIS, Lecture Notes in Computer Science.* vol. 1340, Springer-Verlag, pp. 34–78, 1997.

[13] X. Ren, C. Fowlkes, J. Malik. *Scale-invariant contour completion using conditional random fields.* Proceedings of the 10th IEEE International Conference on Computer Vision, vol. 2, pp. 1214–1221, 2005.

[14] M. de Berg, O. Cheong, M. van Kreveld, M. Overmars. *Computational Geometry: Algorithms and Applications.* Springer-Verlag TELOS Santa Carla, CA, USA, 3rd edition, 2008.

[15] F. P. Preparata, M. I. Shamos. *Computational Geometry – An Introduction.* Springer-Verlag, New York, 1985.

[16] H. Edelsbrunner. *Algorithms in Combinatorial Geometry.* Springer-Verlag, Berlin, 1987.

[17] T. P. Fang, L. A. Piegl. *Delaunay triangulation using a Uniform Grid.* IEEE Computer Graphics and Applications, vol. 13, no. 3, pp. 36–47, 1993.

[18] L. J. Guibas, D. E. Knuth, M. Sharir. *Randomized incremental construction of Delaunay and Voronoi diagrams.* Algorithmica, vol. 7, no. 4, pp. 381–413, 1992.

[19] S. Fortune. *A sweep-line algorithm for Voronoi diagrams.* Algorithmica, vol. 2, no. 2, pp. 153–174, 1987.

[20] L. P. Chew. *Constrained Delaunay triangulations.* Algorithmica, vol. 4, no. 1, pp. 97–108, 1989.

[21] H. Edelsbrunner, R. Seidel. *Voronoi diagrams and arrangements.* Discrete and Computational Geometry, vol. 1, no. 1, pp. 25–44, 1986.

[22] O. Aichholzer, F. Aurenhammer, T. Hackl, B. Jüttler, M. Oberneder, Z. Šír. *Computational and Structural Advantages of Circular Boundary Representation.* Algorithms and Data Structures, 4619, 374–385, 2007.

[23] O. Aichholzer, W. Aigner, F. Aurenhammer, T. Hackl, B. Jüttler, M. Rabl. *Medial axis computation for planar free-form shapes, Computer-Aided Design.* 41, 5, 339–349, 2009.

[24] J. W. Brandt. *Convergence and continuity criteria for discrete approximations of the continuous planar skeleton.* CVGIP: Image Understanding, 59(1), 116–124, 1994.

[25] J. Giesen, B. Miklos, M. Pauly. *The Medial Axis of the Union of Inner Voronoi Balls in the Plane.* Computational Geometry – Theory and Applications, 45, 515–523, 2012.

[26] N. Amenta, M. Bern, D. Eppstein. *The Crust and the β-Skeleton: Combinatorial Curve Reconstruction.* Graphical Models and Image Processing, Vol. 60(2), 125–135, 1998.

[27] N. Amenta, S. Choi, R. Kolluri. *The power crust, unions of balls, and the medial axis transform.* Computational Geometry: Theory and Applications, 19(2-3), 127–153, 2001.

[28] N. Amenta, M. Bern. *Surface Reconstruction by Voronoi Filtering.* Discrete and Computational Geometry, 22, 481–504, 1999.

[29] M. Dobovišek. *Matematika 2.* Izbrana poglavja iz matematike in računalništva (48), DMFA, Ljubljana, 2013.

[30] C. Bär. *Elementary Differential Geometry.* Cambridge University Press, 2010.

[31] L. Piegl, W. Tiller. *The NURBS Book.* 2nd Edition, Springer, 1995.

[32] D. Šurc. *Delaunayeve triangulacije in Voronoievi diagrami.* Delo diplomskega seminarja, Fakulteta za matematiko in fiziko, Univerza v Ljubljani, 2011.

[33] G. Smogavec. *Aproksimacija srednje osi enostavnih mnogokotnikov.* Diplomsko delo, Fakulteta za elektrotehniko, računalništvo in informatiko, Univerza v Mariboru, 2009.

[34] V. Domiter. *Algoritmi triangulacije s strategijo prebiranja.* Doktorska disertacija, Fakulteta za elektrotehniko, računalništvo in informatiko, Univerza v Mariboru, 2009.

[35] K. Sugihara. *Voronoi Diagrams.* V knjigi: G. Farin, J. Hoschek, M. S. Kim. *Handbook of Computer Aided Geometric Design*, Elsevier Science B. V., Amsterdam, 429–450, 2002.

[36] H. I. Choi, C. Y. Han. *The Medial Axis Transform.* V knjigi: G. Farin, J. Hoschek, M. S. Kim, Handbook of Computer Aided Geometric Design, Elsevier Science B. V., Amsterdam, 451–471, 2002.

[37] B. Miklos, J. Giesen, M. Pauly. *Discrete Scale Axis Representations for 3D Geometry.* ACM Transactions on Graphics 29(4), 101:1–101:10, 2010.

[38] B. Sadri, J. Giesen, E. Ramos. *Medial Axis Approximation and Unstable Flow Complex.* Dostopno na: *http://www.cs.toronto.edu/~sadri/presentations/SoCG06.pdf*, dostop 15.02.2014.

[39] H. I. Choi, S. W. Choi, H. P. Moon. *Mathematical theory of medial axis transform.* Pacific Journal of Mathematics, Vol. 181, No. 1, 57–88, 1997.

[40] S. Lambach. *Voronoi and Delaunay diagrams.* Diplomsko delo, Fakultät für Mathematik, Technische universität München, 2010.

[41] B. Gärtner, M. Hoffmann. *Computational Geometry.* Dostopno na: *http://www.ti.inf.ethz.ch/ew/courses/CG13/lecture/cg-2013-.pdf*, dostop 15.02.2014.

[42] J. Heinonen. *Lectures on Lipschitz analysis.* Rep. Univ. Jyväskylä Dept. Math. Stat. 100, 1–77, 2005.

[43] J. Fisher. *Visualizing the Connection Among Convex Hull, Voronoi Diagram and Delaunay Triangulation.* Master's Thesis, Department of Computer Science, Michigan Technological University, Houghton, 2004.

[44] S. W. Choi, S.-W. Lee. *Stability Analysis of Medial Axis Transform under Relative Hausdorff Distance*. Center for Artifical Vision Research, Korea University, 2000.

[45] D. Shaked, A. M. Bruckstein. *Pruning medial axis*. Computer Vision and Image Understanding, vol. 69, no. 2, pp. 156–169, 1998.

[46] M. P. Deseilligny, G. Stamon, C. Y. Suen. *Veinerization: A new shape descriptor for flexible skeletonization*. IEEE Trans. Analysis and Machine Intelligence, vol. 20, no. 5, pp. 505–521, 1998.

[47] M. Held. *On the computational geometry of pocket machining*. Vol. 500 of Lecure Notes in Computer Science, Springer-Verlag, 1991.

[48] D. T. Lee. *Medial axis transform of a planar shape*. IEEE Transactions on Pattern Analysis and Machine Intelligence, PAMI-4, 363–369, 1982.

[49] D. T. Lee, R. L. Drysdale. *Generalization of Voronoi diagrams in the plane*. SIAM Journal of Computing, 10, 73-87, 1981.

[50] V. Srinivasan, L. R. Neckman. *Voronoi diagram for multiply-connected polygonal domains* I : *Algorithm*. IBM Journal of Research and Developement, 31(3), 361–372, 1987.

[51] C. K. Yap. *An* $\mathcal{O}(n\log(n))$ *algorithm for the Voronoi diagram of a set of simple curve segment*. Discrete and Computational Geometry, 2(4), 365–393, 1987.

[52] H. I. Choi, S. W. Choi, H. P. Moon, N. S. Wee. *New Algorithm for Medial Axis Transform of Plane Domain*. Department of Mathematics, Seoul National University and Department of Industrial Engineering, Hansung University, Korea, 1997.

[53] F. Aurenhammer, R. Klein. *Voronoi Diagrams*. V knjigi: J. Sack, J. Urrutia. *Handbook of Computational Geometry*. Elsevier North-Holand, Amsterdam, 201–292, 2000.

[54] H. I. Choi, S. W. Choi, C. Y. Han, T.-W. Kim, S.-H. Kwon, H. P. Moon, K. H. Roh, N.-S. Wee. *Two-dimensional offsets and medial axis transform*. Springer Science, Business Media B. V., 2007.

[55] H. I. Choi, S. W. Choi, C. Y. Han, H. P. Moon, K. H. Roh, N.-S. Wee. *Two-dimensional offsets via medial axis transform* I : *Mathematical theory.* 2001.

[56] M. Kandič. *Problem umetnostne galerije.* Dostopno na: `http://www.fmf.uni-lj.si/~skreko/DodatnoStudijskoGradivo/UmetnostneGalerije/MarkoKandic-ArtGalery.pdf`, dostop 15.02.2014.

[57] T. Grajzar. *Ravninski grafi in izrek Kuratowskega.* Dostopno na: `http://www.fmf.uni-lj.si/~skreko/Pouk/dm2/2005-6/Seminarske/TinaGrajzar_RavninskiGrafi.pdf`, dostop 15.02.2014.

[58] N. Štamcar. *Totalna ukrivljenost krivulj v prostoru in Fáry-Milnorjev izrek.* Diplomsko delo, Pedagoška fakulteta, Fakulteta za matematiko in fiziko, Univerza v Ljubljani, 2012.

[59] R. Fabbri, L. F. Estrozi, L. da F. Costa. *On Voronoi Diagrams and Medial Axes.* Journal of Mathematical Imaging and Vision, 17:1, 27–40, 2002.

[60] M. Pugelj. *Konveksne množice in poliedri.* Dostopno na: `http://www.fmf.uni-lj.si/~juvan/Racunalnistvo3/0809/gradivo/konveksnost.pdf`, dostop 15.02.2014.

[61] *Krivulje v ravnini in prostoru.* Dostopno na: `http://ucilnica1213.fmf.uni-lj.si/pluginfile.php/6973/mod_resource/content/1/krivulje12.pdf`, dostop 15.02.2014.