Povzetek

V današnjem svetu so tako ljudje kot stvari vse bolj povezane med seboj (družbena omrežja, internet, omrežja stvari,...). Zaradi močne povezanosti so ta omrežja lahko zelo dovzetna tako za pretok informacij, kot tudi okužb. V primeru neozdravljive hitro nalezljive okužbe (recimo požar, bolezen, računalniški virus, itd) bi se ta lahko hitro razširila po celotnem omrežju. Za vse vrste okužb bomo v diplomskem delu uporabili izraz *požar*. Če se vozlišče zaščiti pred požarom bomo rekli, da *gasilec v tem vozlišču pogasi požar*.

Požar se širi s prehajanjem od okuženih vozlišč na tiste, s katerimi so ta v stiku (so jim sosednja). Njihova srečevanja lahko ponazorimo z grafi, v katerih točke predstavljajo ljudi, povezave pa stike med njimi. V primeru, ko je bolezen neozdravljiva in zanjo ni cepiva, se najprej počasi, a vztrajno in vse hitreje širi, dokler ne zajame celotne populacije, razen določenega deleža ljudi, ki so morda nanjo odporni. Če imamo možnost še neobolele osebe cepiti in jih s tem zaščititi pred boleznijo, lahko bolezen omejimo samo na delež populacije. V jeziku teorije grafov temu rečemo *problem gašenja,* v katerem nam obolele osebe predstavljajo goreča vozlišča, cepljene (in s tem imune) osebe pa so zavarovana vozlišča.

Problem gašenja se je preučevalo na različnih tipih grafov. Ko se požar začne v splošnem grafu, situacijo prevedemo na enega že poznanih in rešenih variant. V diplomskem delu bom prikazal nekatere od njih, kot so: drevesa, zunanje ravninski grafi, končne in neskončne pravokotne mreže, pasovne mreže, šestkotniške mreže, kamor gasilec ali več njih lahko prispe že ob začetku požara, je tam postavljen že pred požarom ali šele po končno mnogo korakih širjenja požara.

Abstract

In the world of today people and things are increasingly becoming interconnected (social networks, internet, networks things etc.). Due to strong interconnection, these networks are very susceptible for the flow of information but also for diseases. In case of incurable very contagious infection (this could be fire, disease or a computer virus, etc.) the latter could easily spread all over the entire network. In this diploma thesis, we shall use the expression "fire" for all these kinds of diseases. If the node is protected against fire, we shall say, the firefighter in this node extinguishes the fire.

The fire is spread by transition from the infected nodes to the ones the infected ones are in connection with (they are their neighboring ones). Their meetings could be depicted by graphs where points are representing the people and connections their contacts between them. In case when the disease is incurable and there is no vaccine, it spreads slowly at first. It is persistent, however, and it spreads faster and faster until it reaches the entire population except for a limited number of people, who are perhaps immune to it. If we have the opportunity to vaccinate the persons who are not yet diseased and protect them against the disease in this way, the disease could be limited only to one part of the population. In the language of the graph theory, we could

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