Povzetek

V delu obravnavamo tri kombinatorične probleme, ki jih rešimo s pomočjo uporabe determinant in permanent matrik. Prvi problem je štetje poti in nesekajočih se poti s predpisanimi začetnimi in končnimi točkami v usmerjenem acikličnem grafu, drugi problem je štetje vpetih dreves (gozdov) v neusmerjenem grafu in tretji problem je štetje razvrstitev števil s predpisanimi padci vrednosti.

Math. Subj. Class. (2010): 11C20, 05C50

Ključne besede: determinanta, permanenta, graf, vpeta drevesa, vpeti gozdovi, permutacija.

Abstract

In the thesis, we study three combinatorial problems that can be solved by applying determinants and permanents of matrices. The first problem refers to counting paths and nonintersecting paths with prescribed initial and final vertices in a directed acyclic graph, the second problem relates to counting the spanning trees (forests) in an undirected graph and the final, third problem, relates to counting the arrangements of numbers with specified descents.

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Keywords: determinant, permanent, graph, spanning trees, spanning forests, permutation.

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